Answers
Re-exam in Public Finance - Fall 2018
3-hour closed book exam

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Part 1: Tax incidence under imperfect competition

Consider a market for good x produced by a monopoly firm with a fixed marginal cost c. The demand for the good implies a willingness to pay given by $p_D(x)$. The government considers imposing either an unit tax (t) in which case $p_D(x) = t + p_S$ or an ad valorem tax (τ) in which case $p_D(x) = (1+\tau)p_S$, where p_S is the pre-tax price set by the firm.

Profit maximization by the monopoly firm implies the following price setting

$$p_S + t = p_D(x) = \frac{c+t}{1-\frac{1}{\varepsilon}} \tag{1}$$

in the case of the unit tax and

$$p_S(1+\tau) = p_D(x) = \frac{c(1+\tau)}{1-\frac{1}{\varepsilon}} \tag{2}$$

in the case of the ad valorem tax, where $\varepsilon = -\frac{dx}{dp_D(x)}\frac{p_D(x)}{x}$ is the elasticity of demand, which is assumed to be constant.

(1A) *Explain the difference between the formal incidence of a tax and the economic incidence.*

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The formal tax incidence describes who has the legal obligation to pay a tax, while the economic incidence describes the economic burden of the tax on the different agents. The economic incidence may differ from the legal incidence if the agents who bear the legal incidence are able to shift the burden to other agents through a change in prices/wages. If for example capital owners have to pay tax on they capital income, they might shift the burden to firms through higher rental rates for capital use.

***(1B)** Derive the effect of an increase in t and τ on the pre- and post tax prices. To what extent is the tax burden shiftet to the consumers in the two cases?*

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The effect of an increase in the unit tax (t) on p_D and p_S is obtained by differentiating (1) wrt. t.

$$\frac{dp_D(x)}{dt} = \frac{1}{1-\frac{1}{\varepsilon}} > 1, \qquad \frac{dp_S}{dt} = \frac{dp_D(x)}{dt} - 1 = \frac{\frac{1}{\varepsilon}}{1-\frac{1}{\varepsilon}} > 0.$$

Similarly, the effect of an increase in the ad valorem tax (τ) is obtained by differentiating (2) wrt. τ.

$$\frac{dp_D(x)}{d\tau} = \frac{c}{1-\frac{1}{\varepsilon}} = p_S, \qquad \frac{dp_S}{d\tau} = \frac{d\frac{p_D(x)}{1+\tau}}{d\tau} = 0.$$

In the case of the unit tax, we see that a marginal increase in the tax also lead to an increase in the pre-tax price (p_S) and hence, a more than proportional increase in the post-tax price (p_D). In order words, with constant marginal costs c of production and a constant and finite demand elasticity, unit taxation leads to overshifting of the tax burden to consumers, when the market is supplied by a monopoly.

In the case of the ad valorem tax, the pre-tax price (p_S) is unaffected by a tax increase, while the increase in the post-tax price (p_D) is proporational to the pre-tax price. Hence, with an ad valorem tax there is proportial shifting of the tax burden to consumers and hence less shifting than with an unit tax.

***(1C)** Compare the pre- and post tax prices with ad valorem taxation to the pre- and post tax prices with unit taxation in the case where $t = \tau c$. Would the monopoly firm prefer unit or ad valorem taxation in this case? Explain why.*

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Starting with the post-tax prices, we see that

$$p_D^{Unit} = \frac{c+t}{1-\frac{1}{\varepsilon}} = \frac{c(1+\tau)}{1-\frac{1}{\varepsilon}} = p_D^{Ad\,valorem},$$

when $t = \tau c$. As the port-tax prices are the same, so are also the equilibrium demand for the good and hence the total number of goods sold. However, looking at the pre-tax prices we see that

$$p_S^{Unit} = \frac{c+t}{1-\frac{1}{\varepsilon}} - t = \frac{c+t\frac{1}{\varepsilon}}{1-\frac{1}{\varepsilon}} > \frac{c}{1-\frac{1}{\varepsilon}} = p_S^{Ad\,valorem}.$$

With the unit tax, the monopoly will in other words set a higher pre-tax price, which (given the same number of goods sold) gives the monopoly higher profits. Consequently, the monopoly firm prefers unit taxation over ad valorem taxation. A similar argument can be made based on the government's tax revenue, which is higher in the case of ad valorem taxation. As the total value of trade ($x \cdot p_D$) is the same with unit taxation as with ad valorem taxation, the difference tax revenue directly mirrors the difference in monopoly profits in the two cases.

The monopoly firm sets a lower pre-tax price in the case of ad valorem taxation, because the effective tax in the ad valorem case depends on the pre-tax price of the monopoly firm (higher p_S also increases τp_S). The monopoly firm is therefore "punished" more for price increases with ad valorem taxation than with unit taxation, which gives an incentive to keep a lower pre-tax price.

Part 2: Tax evasion

Below we consider three different models of tax evasion. In these three models, taxpayers are assumed to maximize the expected utility denoted by U^e. The model equations are

$$\begin{array}{lll l}
\textit{All models} & x^{nc} & = (1-t)Y + tE & (A) \\
\textit{All models} & x^{c} & = (1-t)Y - FtE & (B) \\
\textit{Model 1} & U^e & = (1-p)x^{nc} + px^c & (C) \\
\textit{Model 2} & U^e & = (1-p)u(x^{nc}) + pu(x^c) & (D) \\
\textit{Model 3} & U^e & = (1-p(E))\,x^{nc} + p\,(E)\,x^c & (E)
\end{array}$$

where Y is true income, x is consumption, t is the tax rate, E is unreported income, p is the probability of being detected, F is a fine in proportion to the evaded tax and $u(\cdot)$ is a positive and strictly concave utility function of consumption ($u' > 0, u'' < 0$).

(2A) *Provide a definition of tax evasion, and describe how tax evasion differs from tax avoidance.*

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Tax evasion is defined as a <u>legal</u> and <u>taxable</u> economic activity, not declared to the tax authorities. Hence, the reduction in tax liability is illegal. Examples of tax evasion are underreporting of income or overstating deductions on the tax return. Tax avoidance is also a reduction in tax liability, but it is legal and reflects "tax planning" not intended by the policy makers. Examples of tax avoidance is shifting of income from high tax years to low tax years (for example following a tax reform).

It may be mentioned that shadow/hidden economy activities include tax evasion, but also illegal economic activities, where payments are made but not reported to the tax authorities. It may also be mentioned that the unmeasured economy covers the shadow economy plus do-it-yourself activities.

(2B) *Provide an economic interpretation of the contents in each of the five equations in (A)-(E).*

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Equation (A) defines the consumption if not caught evading. It is equal to the income after tax when reporting truthfully (the first term) and the taxes saved by evading the amount E (the second term). Equation (B) defines the consumption if caught evading. It is equal to the income after tax when reporting truthfully (the first term) and the fine the taxpayer has to pay from the detected evasion, which equals F times the evaded tax payment.

Equation (C) states that the taxpayers maximize expected utility, which in model 1 is equivalent to expected income (i.e., an implicit assumption is that the agent is risk neutral corresponding to utility being linear in income). The first term in the equation is the consumption if not caught evading multiplied by the probability of not being caught, while the second term is the consumption if caught evading multiplied by the probability of being caught. Equation (D) states that in model 2 the taxpayers maximize expected utility, which is the same as in model 1 with the exception that the taxpayers now have declining marginal utility of consumption (captured by the concave utility function $u(\cdot)$). Hence, in model 2 taxpayers are risk averse. Equation (E) states that in model 3 the taxpayers maximize expected utility, which is the same as in model 1 with the exception that the probability of being caught is a function of the amount evaded.

(2C) *Show that taxpayers in model 1 will evade taxes if and only if*

$$(1-p)t - pFt > 0. \tag{3}$$

Provide an economic interpretation of this result. How does the size of p, t and F affect the incentive to evade?

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The optimal behavior of a taxpayer in model 1 is found by inserting equations (A) and (B) into equation (C) and differentiating wrt. E. After inserting equations (4) and (5) into equation (1), we have

$$U^e = (1-p)\left((1-t)Y + tE\right) + p\left((1-t)Y - FtE\right) = (1-t)Y + (1-p)tE - pFtE.$$

Differentiation wrt. E gives

$$\frac{dU^e}{dE} = (1-p)t - pFt.$$

If this is positive, the taxpayer will evade taxes (because of the linear structure the taxpayer will evade on all income ($E = Y$)), and if it is negative then the taxpayer will not evade. The first term in the expression is the marginal benefit of evading one additional euro equal to the increase in net-income if not caught, while the second term is the marginal cost reflecting the increase in the fine paid if caught. A higher probability (p) of being caught will increase the marginal costs and reduce the marginal benefits and thereby reduce the incentive dU^e/dE . A higher fine F increases the marginal costs of being caught and thereby reduce the incentive dU^e/dE. A higher tax rate t does not affect whether the incentive dU^e/dE is positive or negative, and does therefore not affect the decision to evade or not (if $(1-p) - pF$ is positive then a higher t will make dU^e/dE more positive and vice versa).

In model 2 you can show that taxpayers will increase tax evasion as long as

$$\frac{dU^e}{dE} = (1-p)u'(x^{nc})t - pu'(x^c)Ft > 0. \tag{4}$$

(2D) *Provide an economic interpretation of this result. Does model 2 predict more or less tax evasion that model 1? Explain why.*

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Equation (4) is similar to the first order condition in question (2C) with the addition of the marginal utilities of consumption in the two states (not caught and caught). As the taxpayers in this model is risk adverse, their marginal utility of consumption decreases with the consumption levels. Hence, the more a taxpayer evade the larger is the difference between the marginal utilities of consumption between the two states. In the state where she is not caught, the consumption level is higher and the marginal utility of consumption lower. The addition of risk adversion therefore implies that the marginal benefits of evading taxes is decreasing in the amount evaded, while the marginal costs are increasing. This implies that taxpayers (in general) nolonger will evade on all income, but choose an interier solution. If taxpayers evaded taxes in model 1, model 2 therefore predict less tax evasion.

Hower, model 2 does not change evasion at the extensive margin. Evaluated at $E = 0$,

$x^{nc} = x^c \Rightarrow u'(x^{nc}) = u'(x^c)$. Hence, if the first order condition in model 1 is positive, so it is at $E = 0$ in model 2, implying that everybody will evade at least a small part of their taxes.

In model 3 you can show that taxpayers will increase tax evasion as long as

$$\frac{dU^e}{dE} = (1 - p(E))\,t - p(E)\,tF - p'(E)\,(x^{nc} - x^c) > 0 \tag{5}$$

(2E) *Provide an economic interpretation of this result. Assuming that $p'(E) > 0$, does model 3 predict more or less tax evasion that model 1? Why should we expect that $p'(E) > 0$ in Denmark (and many other countries)?*

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Equation (5) is similar to the first order condition in question (2C) with the addition of the third term $p'(E)\,(x^{nc} - x^c)$. The third term is an additional marginal cost of evading taxes that depends, not on the level of p, but on the change in p when E is increased. If p increases with E, the taxpayer needs to take into account that more evasion increases the risk that all of the income evaded so far will be detected, and thereby triggering a drop in consumption of $x^{nc} - x^c$. This additional marginal cost reduces tax evasion compared to evasion level in model 1.

Hower, similar to model 2, model 3 does not change evasion at the extensive margin. Evaluated at $E = 0$, $x^{nc} = x^c$. In this case equation (5) collapses to the first order condition in model 1. Hence, if the first order condition in model 1 is positive, so it is at $E = 0$ in model 3, implying that everybody will evade at least a small part of their taxes.

One reason for why we should expect $p'(E) > 0$ in Denmark is that the tax authorities already have some information about the individual taxpayer through third party reporting from e.g. her employer, bank etc. This implies that if the taxpayer starts reporting less income than the income reported by her employer, the tax authorities will automatically know that the taxpayer is evading and perform an audit.

Part 3: The elasticity of taxable income

Consider individuals with preferences represented by the utility function

$$u(c, z) = c - \frac{1}{1 + \frac{1}{\varepsilon}} z^{1 + \frac{1}{\varepsilon}}, \tag{6}$$

where c is consumption, z is labor supply and ε is a preference parameter. The budget constraint is given by

$$c = (1 - t)z. \tag{7}$$

(3A) *Illustrate in a diagram with z on the primary axis and c on the secondary axis the initial optimum of an individual. How does the optimum change if the tax rate is increased from t_1 to $t_2 > t_1$? Comment on the directions of the income and substitution effects.*
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The initial optimum and the effect of the tax change is illustrated in figure 1 below.

– Figure 1 –

The substitution effect is driven by the change in the (marginal) net-of-tax rate and with standard preferences a higher marginal tax rate will decrease labor supply. The income effect is driven by the mechanical change in disposable income folloing a tax change. A higher tax rate descreases disposable income and if leasure is a normal good, the individuals will respond by consuming less and thereby increase labor supply.

In the case here preferences are quasi-linear and the income effect is therefore zero.

(3B) *Give the intuition for why it might be more correct to look at the change in taxable income when computing the marginal deadweight loss instead of just hours worked?*
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A tax may affect behavior in a number of dimensions other than just hours worked. E.g. a higher tax might reduce the willingness to accepted a higher paying job further away or give a higher incentive to transform earns into fringe benefits (better coffee machines etc.). Behavioral responses across all of these dimensions cause distortions that should be included in a calculation of the marginal deadweight loss. However, instead of analysing each of all these potential margins separately, they are all captured by the change in taxable income (less willingness to travel, more fringe benefits etc. all reduce taxable income). As argued by Martin Fledstein (1995, 1999) analysing the changes in taxable income is therefore sufficient when calculating the (marginal) deadweight loss. The elasticity of taxable income (ETI) is therefore often called a sufficient statistic.

The article "The Effect of Marginal Tax Rates on Taxable Income: A Panel Study of the 1986 Tax Reform" in the Journal of Political Economy (1995) by Martin Feldstein studies the effect of the 1986 tax reform on the taxable income reported by different income groups. The reform significantly reduces marginal tax rates while broadering the tax base. Below is a copy *of Table II from the article showing the main estimate from the paper.*

(3C) *Describe the empirical analysis and explain, using Table 2 below, how the author arrives at the estimates of the implied elasticity of taxable income (ETI). What are the main identifying assumptions needed for the estimates to be the causal effect of the marginal tax rates on taxable income?*

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Feldstein (1995) estimates the effect of changes in the marginal tax rate on taxable income using a difference-in-differences (DiD) estimation framework. However, contrary to "normal" DiD estimates, he does not have an untreated control group but rather different groups with different treatment intensities. Here the treatment intensity rises with income. I.e. high income groups experience the largest changes in their marginal tax rate.

In Table II we see the simple DiD estimation table. Denote the precentage change in taxable income of group i from before to after the reform as $\Delta log(E^i)$, where i can be Medium (M), High (H1) or Highest (H2). Similar denote the precentage change in the net-of-tax income $(1-t)$ as $\Delta log(1-t^i)$. Using this, we can compute the implied elasticity in row 7 (High minus medium) by

$$ETI = \frac{\Delta log(E^{H1}) - \Delta log(E^M)}{\Delta log(1-t^{H1}) - \Delta log(1-t^M)} = \frac{21,0-6.2}{25,6-12.2} = 1.10,$$

which is the DiD estimate. The estimate has the expected sign as the groups that experienced the largest falls in their marginal tax rate had the largest increases in their taxable income. However, the size of the implied elasticities are much larger than modern studies show today.

For the estimates to be causal we need two assumptions:

- The common/parallel trend assumption, which states that, in the absence of the reform, the change in the taxable income for the three groups should be the same.
- Same underlying elasticity for all groups. I.e. if the groups had gotten the same change in the net of tax rates, we would have see the same change in taxable income for all groups.

If one of these assumptions are invalid the estimates will be biased.

(3D) *Describe how you could validate the main identifying assumptions needed in (3C) and what kind of data you would need to do so.*

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One way to validate if the common trend assumption seems plausible is to consider the evolution of the taxable income of the different groups over longer time periods before and/or after the reform. If the taxable incomes move in parallel in these non-reform years it speaks to the validity of the common trend assumption. More formally this can be tested by running the DiD estimations in non-reform years. The estimates from these "Placebo tests" should be insignificant.

The assumption of the same underlying elasticity for all groups is more difficult to validate. Here you would need other reforms, where all groups were treated to the same extend and see if taxable incomes move in parallel in these years or reforms with untreated control groups.

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TABLE 2

ESTIMATED ELASTICITIES OF TAXABLE INCOME WITH RESPECT TO NET-OF-TAX RATES

Taxpayer Groups Classified by 1985 Marginal Rate	Net of Tax Rate (1)	Adjusted Taxable Income (2)	Adjusted Taxable Income Plus Gross Loss (3)
	Percentage Changes, 1985–88		
1. Medium (22–38)	12.2	6.2	6.4
2. High (42–45)	25.6	21.0	20.3
3. Highest (49–50)	42.2	71.6	44.8
	Differences of Differences		
4. High minus medium	13.4	14.8	13.9
5. Highest minus high	16.6	50.6	24.5
6. Highest minus medium	30.0	65.4	38.4
	Implied Elasticity Estimates		
7. High minus medium		1.10	1.04
8. Highest minus high		3.05	1.48
9. Highest minus medium		2.14	1.25

NOTE.—The calculations in this table are based on observations for married taxpayers under age 65 who filed joint tax returns for 1985 and 1988 with no age exemption in 1988. Taxpayers who created a subchapter S corporation between 1985 and 1988 are eliminated from the sample.